Conductivity- Depth Relation for the SSFL

1.0 Overview
Snow (1968; 1970) and USGS (Belcher, 2001; D’Agnes, 1997) have observed that bulk hydraulic conductivities in fractured rock tend to decline rapidly with depth. The reduction in conductivity with depth is due to the increased closure of fractures with depth due to the increasing normal stress from the weight of the overlying rock. Empirical relations of rock joint deformation under variable stress were developed experimentally by Bandis et al. (1983). His experiments were developed to understand the potential for rock movement (deformation / differential settlement) due to the weight of structures erected on top of rock formations. His work was published in the International Journal of Rock Mechanics and has been cited in subsequent literature.

The USGS work in Death Valley, Nevada (D’Agnese, 1997) observed a conductivity decrease with depth as is postulated and observed at the SSFL. D’Agnese (1997) indicated qualitatively that the hydraulic conductivity in fractured rock at depths between 300 -1000 m decreases rapidly; beyond that depth, the hydraulic conductivity was suspected to be dominated by the rock matrix. Through their investigations they attempted to correlate field observed hydraulic conductivity measurements to a depth relation. They found that there was a “significant relation between depth and log10 transformed hydraulic conductivity” through statistical analyses, however they also found that “estimates can still vary considerably at a given depth”. As a result, the USGS stopped short of developing an equation for predicting depth dependent hydraulic conductivity for the Death Valley modeling. The USGS has however developed a depth-dependence function that is included in MODFLOW 2000 (Anderman and Hill, 2003). The USGS work supports the use of a conductivity-decrease-with-depth relation.

Recognizing that the weight of the overlying rocks can close or significantly reduce fracture apertures at the SSFL, we initially evaluated and applied the relation of fracture closure with depth using the equation developed by Bandis et al. (1983) for a sandstone sample from the UK. This relation was further refined to reflect the conditions at SSFL by considering the effects of fracture dip angles, confining stresses, porewater pressure, and fracture aperture distribution at the SSFL.

Subsequently field data was collected (Cheema et al., 2006) to determine the joint/fracture properties of sandstone and shale samples from the SSFL. These properties were used to develop site-specific stress-closure relations for the SSFL. The Barton-Bandis model (Barton et al., 1985; Barton and Bakhtar 1987) utilizes measurements fracture/joint properties to extend the observations by Bandis et al. (1983) for prediction of stress-closure for rock samples from other sites. The Barton-Bandis model was applied at the SSFL to further refine the conductivity-depth relation to a site-specific relation for sandstone and shale.
2.0 \hspace{1em} \textbf{The Bandis et al. Sandstone Conductivity-Depth Relation}

In the paper by Bandis et al. (1983) the methodology use to deriving relations for predicting fracture closure with depth (normal stress) for a number of rock samples (sandstone, siltstone, shale) in the UK is presented.

The empirical equation developed by Bandis et al. (1983) was initially applied to the SSFL to estimate the fracture aperture closure with an increasing normal stress using physical characteristics of the UK sandstone sample in the Bandis et al. (1983) paper. The empirical relation derived for a sandstone unit was:

\[ \log \sigma_n = -0.758 + 6.987 \Delta V_j \]

Where: \( \sigma_n \) = normal confining stress in MPa; and \( V_j \) = joint closure in mm

This equation was developed experimentally by loading and unloading field samples of fractured rock for natural, fresh and weathered, rough-walled fractures with “mismatched” joints. Bandis et al. developed these equations by testing samples of six types of rock including one sandstone, two types of slate, two types of limestone and one siltstone. The sandstone samples tested are described as fresh to weathered. All samples tested contained a single fracture (joint plane) aligned parallel to the loading ends (so the load applied was the normal stress). Figure 1 presents the experimental joint closures and the empirical fit of the data from Bandis et al. (1983). The best-fit equation for sandstone (SDST. No. 2) is presented on this figure.

\textbf{Figure 1:} Semi-log plots of normal stress vs. closure of mismatched joints. (From Figure 13 of Bandis et al. (1983))

\hspace{1em} AquaResource Inc, 2007.
To apply this equation for the SSFL the equation was re-arranged to solve for the joint closure ($V_j$) based on the computed static stress due to the weight of the overlying rock mass. The resultant equation is as follows:

$$\Delta V_j = \frac{\log \sigma_n + 0.758}{6.987}$$

For the calculation of the normal stress, it was assumed that the weight of the rock mass, which acts vertically downward, was normal to the fracture set. Normal stress was calculated for depth intervals referred to as “depth bins”. Using the above equation the fracture closure can be computed for each depth by specifying the average surface aperture. An average surface aperture of 150 $\mu$m was used based on mean aperture at a depth of 50-100 m of 10 $\mu$m from field data (Montgomery Watson, 2000).

The average bulk conductivity at the 50 m depth (average depth of pumping test wells) was assumed to be $10^{-7}$ m/s and the fracture aperture computed at this depth is 10 $\mu$m. Multiplying these together provides us with a K coefficient of 4.4E-5. This coefficient is multiplied by the fracture aperture for each depth bin to compute bulk hydraulic conductivity.

For application within the Mountain Scale Groundwater Flow Model (MSGFM), the ranges of hydraulic conductivity were computed divided into “depth bins”. A representative multiplier was assigned to each depth bin. This initial relation is shown on Figure 3 and is referred to as the “2005 Representation.” Application within the model is completed for each element by multiplying the input hydraulic conductivity by the multiplier for the appropriate depth bin. Input hydraulic conductivities were derived from pumping tests that were typically completed within the 50-100 m depth range.

Assumptions

For the initial application using the Bandis et al. (1983) sandstone, the following assumptions were included:

- Joint/Fracture Properties of the UK Sandstone are sufficiently similar to the SSFL Sandstone
- The system is in static equilibrium, such that no dynamic (including tectonic) forces are active;
- Mean aperture of the sandstone is 150 $\mu$m at a representative depth (~50-100 m), varying from ~250 $\mu$m at ground surface to 50 $\mu$m at depth based on field data from Montgomery Watson (2000);
- Analysis of the conductivity change with depth is based on the closure of the mean aperture (the variability about the mean is not considered);
- Joint closure initiates at ground surface and extends to some threshold depth where maximum joint closure is approach $V_{max}$ as described by Bandis et al. (1983), and as suggested by on-site observations and USGS observations at Death Valley;
• Joint closure occurred at the same rate for bedding parallel and bedding-perpendicular fractures;
• Normal stress could be approximated by the weight of the overlying rock mass;
  o Angle of the bedding did not affect the normal stress;
  o Pore water pressure did not affect the normal stress;
• Normal stress was not significantly impacted at the edge of the mountain, even though the confining pressure would be reduced at the edge of the mountain; and
• The strength of infilling material within the joints is similar in strength and physical properties to the surrounding sandstone.

Each of these assumptions may limit the applicability of the computed conductivity-decrease-with-depth relation for the MSGFM. Existing site information, including USGS hydrophysical logging (MWH, 2007) and interpretation of site water levels suggest that the initial application of the conductivity-decrease-with-depth relation needs additional refinement to provide site specific estimates for the SSFL.

3.0  Modification of Bandis et al. Relation for the SSFL

In an effort to improve the “representativeness” of this relation for the SSFL, the following initiatives were undertaken:

1) Calculation of normal stress adjusted to reflect the average bedding angle and the confining stress occurring due to expansion of the adjacent confined rock mass (expressed through Poisson’s ratio);
2) Reduction of the normal stress due to the reduction of the confining stress near the margins of the mountain;
3) Consideration of the pore water pressure in reducing the normal stress compressing the fracture;
4) Consideration of the suite of fractures (varying apertures) and how the range of apertures changes with depth; and

Site-Specific Refinements
1) Normal Stress
   a) Confining Stress due to Poisson’s Ratio
   Poisson’s ratio (ν) for Sandstone is approximately 0.04 to 0.05 (Davis & Reynolds, 1996, p.145), which yields a horizontal stress at depth as follows:

   \[ \sigma_x = \frac{\nu}{1-\nu} \sigma_z = \frac{0.05}{0.95} \sigma_z \approx 0.05 \sigma_z \]

   Thus for Sandstone, the confining stress due to Poisson’s ratio is expected to be minor relative to the stress from the weight of the overlying rock mass. This would not be the case for limestone, since it has a higher ratio (0.11 to 0.25).

   b) Stress Rotation to Account for Dipping Bedding Planes
   To obtain the stress value normal to the plane of the fractures we must use both principle stresses (stress due to the weight of the overlying rock (\(\sigma_z\)) and the
confining stress(σ_y) based on the following stress relation (Davis & Reynolds, 1996).

\[ σ_n = \frac{σ_x + σ_y}{2} + \frac{σ_z - σ_y}{2} \cos 2θ \]

where θ is the rotation angle between the axis of the principle stress and the normal to the plane. At SSFL, θ ~ 60° (or –30°). This results in a normal stress that is approximately 76% of the magnitude of the vertical confining stress (2005 Representation).

2) Reduction in Confining Stress Near Edge of Mountain

To evaluate the reduction in confining stress near the edge of the mountain, let’s assume that the confining stress (σ_y) trends to 0. The result is that the weight of the overlying material is the only principle stress (essentially the same as that assumed previously). Allowing for the stress rotation as above, the normal stress in this case becomes approximately 75% of the vertical confining stress (2005 Representation).

3) Effective Stress

The effective stress at a point in the subsurface is the difference between the total normal stress (imparted by the weight of the rock overlying that point) and the pore water pressure (or buoyancy force). The relation is \( \sigma' = \sigma - \mu \), where \( \sigma' \) is the effective stress and \( \mu \) is the pore water pressure. The pore water pressure (force) is calculated as the product of the water density, gravitational constant and the height of the water column above that point (essentially 9.81 kPa/m * height of water column).

The total stress due to the weight of the overlying rock is approximately 24.1 kPa/m * the depth. If you assume the rock is fully saturated, the effective stress is thus 24.1 – 9.81 = 14.29 kPa/m * the depth. Resulting in a 40% reduction in the conductivity-decrease-with-depth relation compared to the 2005 Representation.

The following table provides a comparison between the previous (2005 Representation) and revised conductivity decrease with depth factors referred to as “1 Representative Aperture” on Figure 3.

<table>
<thead>
<tr>
<th>K Factors used in “2005 Representation”</th>
<th>Revised K Factors“1 Representative Aperture”, Accounting for Bed Angle, Confining Stress, Porewater Pressure</th>
<th>% Higher</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Depth (m) To Depth (m) Depth Factor (multiplier)</td>
<td>From Depth (m) To Depth (m) Depth Factor (multiplier)</td>
<td></td>
</tr>
<tr>
<td>0 20 1</td>
<td>0 20 2.6667</td>
<td>167%</td>
</tr>
<tr>
<td>20 40 1</td>
<td>20 40 2.05</td>
<td>105%</td>
</tr>
<tr>
<td>40 70 1</td>
<td>40 60 1.4433</td>
<td>44%</td>
</tr>
<tr>
<td>70 80 0.85</td>
<td>60 80 1.1233</td>
<td>32%</td>
</tr>
<tr>
<td>80 90 0.7</td>
<td>80 90 0.95</td>
<td>36%</td>
</tr>
<tr>
<td>90 100 0.6</td>
<td>90 100 0.86</td>
<td>43%</td>
</tr>
<tr>
<td>100 110 0.5</td>
<td>100 110 0.785</td>
<td>57%</td>
</tr>
<tr>
<td>110 120 0.45</td>
<td>110 120 0.72</td>
<td>60%</td>
</tr>
</tbody>
</table>
As this table shows, the hydraulic conductivity within every depth zone increased from the 2005 Representation when the effect of bedding angle, confining stress and porewater pressure were included, with the most significant increases between 140 to 450 m (450 to 1500 ft). This increase in K factors translates into a slower rate of decrease in conductivity with depth. The revised conductivity-decrease-with-depth relation contains a notable decrease within the range of 0 to 300 m (almost 1 order of magnitude) based on the K factor. While a decrease within this range was not acknowledged by D’Agnes et. al. (1997), a similar decrease is evident in the reported graph of conductivity measurements with depth at Death Valley (Figure 2). Thus, the revised relation does appear to be consistent with the USGS observations at Death Valley.

Further, the USGS data suggested that the conductivity decrease continued through the range of 300 to 1000 m, while our current function reaches the matrix value at a depth of about 500 m (threshold depth). Two factors may explain that difference: 1) the hydraulic conductivity is significantly larger in Death Valley than at SSFL (near surface K ~ $10^{-1}$ to $10^{-3}$ cm/s – see Figure 2, included below), and 2) the assumption that conductivity is controlled by the mean aperture skews results toward lower hydraulic conductivity values.

Point 2 is significant. By only using the mean aperture, we reach a point at about 500 m depth where that aperture would be closed, resulting in matrix permeability.
However, considering the likely distribution of apertures at any point, larger aperture fractures would still be active at this depth, and would likely dominate the hydraulic conductivity below 500 m. Step 4 analyzes that affect further.

4) Effect of Considering a Distribution of Apertures

To evaluate the effect that a suite of variable aperture fractures might have in determining the hydraulic conductivity with depth, a fracture aperture probability distribution function was implemented. This probabilistic distribution of apertures was used to assess the effect of fracture closure (and associated hydraulic conductivity). The probability distributions was initially developed for use in SSFL FRACTRAN runs from packer testing. The distribution is a lognormal with a mean aperture of 0.1 mm (100 µm) (Montgomery Watson, 2000). Two values of variance were evaluated: 1) 0.5 m² and 2) 0.2 m². In considering this fracture distribution, only bedding-parallel fractures are considered, and it is assumed that all bedding parallel fractures continue to have productive interconnections (such that they do not become dead-end fractures).

The effect of including this fracture distribution was that even at considerable depths, some fractures remained active (those fractures that were originally largest). This has the effect of reducing the rate of decrease in conductivity throughout the 1000 m interval tested.

The graphs below illustrate the changes to the conductivity-decrease-with-depth relations. Two sets of graphs are provided, one for each variance tested. Each of the graphs contains 4 curves as follows:

I. “2005 representation” – the function applied in the MSGFM model calibration to January 2006;
II. “1 representative aperture” – the revised function resulting from the first 3 considerations discussed (pore water pressure, rotation angle, confining stress);
III. “Mean” - calculation of representative aperture taken as the mean of the open fracture apertures (may skew aperture toward larger fractures, assumes all fracture apertures in probability distribution are equally likely);
IV. “Probability Weighted Average” - calculation of representative aperture taken as the weighted average of the open fracture apertures (probability = weighting function). This approach accounts for the lower probability of having a large fracture at any given depth.

All curves reflect the same type of asymptotic function and are relatively consistent. Functions II through IV suggest high conductivities, particularly throughout depth range from aero to 100 m. Having less reduction within this zone is more consistent with the USGS logging (MWH, 2007) than the function that was previously applied.

**Figure 3:** Computed Relation of Conductivity with Depth for the SSFL. Variance only applicable to the Mean Aperture and Probability Weighted relations.
The graphs in Figure 3 illustrate the “exponential” type of decrease-with-depth that has been adopted by others such as Anderman and Hill (2003).

For approaches III and IV, the aperture closure is used to first calculate the fracture hydraulic conductivity (cubic law), and subsequently to estimate the bulk hydraulic conductivity as presented previously. This step was necessary since fracture conductivity is sensitive to the square of the aperture, and the bulk conductivity at greater depths will contain fewer active fractures. The bulk hydraulic conductivity is estimated based on the ratio of active to total potential fracture volume at a given depth (the total potential volume is given by the area under the probability distribution curve). The difference between the fracture hydraulic conductivity and the calculated bulk hydraulic conductivity is presented in the Figure 4. Throughout the depth range from 100 to 1000 m shown on Figure 5, fractures with a similar magnitude of hydraulic conductivity are expected to occur. However, the frequency of occurrence is expected to decrease with depth, thus the bulk hydraulic conductivity continues to decrease with depth. The bulk hydraulic conductivity is the condition used in the MSGFM which represents the fractured rock as an equivalent porous medium.

**Remaining Assumptions / Limitations**

The following assumptions / limitations remain:

- Joint/Fracture Properties of the UK Sandstone are sufficiently similar to the SSFL Sandstone
- The system is in static equilibrium, such that no dynamic (including tectonic) forces are active;
- Joint closure initiates at ground surface and not below some threshold depth, as suggested by on-site observations and USGS observations at Death Valley;
- Joint closure occurs at the same rate for bedding parallel and bedding-perpendicular fractures.
- Joint closure within bedding-perpendicular fractures does not limit the potential for fractures at depth to yield water (interconnection is not interrupted at depth);
- Normal stress was not impacted at the edge of the mountain, even though the confining pressure would be reduced at the edge of the mountain by 25%; and
- The strength of infilling material within the joints is similar in strength and physical properties to the surrounding sandstone.

These assumptions limit accuracy of the conductivity-decrease-with-depth prediction for the SSFL Mountain Scale Model. However, the relation better represents the physical conditions and appears to be more consistent with qualitative field observations (e.g. Geophysics).
4.0 Application of the Barton-Bandis Model for SSFL Specific Conductivity-Depth Relation

In the paper by Bandis et al. (1983) the methodology for estimating site specific fracture closure with depth (normal stress) from rock samples is presented. Subsequent papers by Barton et al. (1985) and Barton and Bakhtar (1987) extend the observations by Bandis et al. (1983) for prediction of stress-closure for rock samples from other sites. They present methodologies to characterize joint and fracture properties for input into an empirical model for predicting fracture closure with changes in stress based on the Bandis et al. (1983) experiments. The model they use is referred to as Barton-Bandis model which is applied in this project to compute effective normal stress-fracture closure and conductivity curves. Cheema et al. (2006) provide a review of the literature describing the Barton-Bandis model and its applicability to SSFL.

In the absence of a methodology for directly measuring fracture closure with depth, the Barton-Bandis model provides a method that utilizes physical measurements that can be made directly from rock core samples to predict stress-closure. The following properties of fractures can be measured from a given rock sample:

1. Compressive strength ($\sigma_c$);
2. Joint Compressive Strength (JCS);
3. Joint Roughness Coefficient (JRC) - via tilt tests or straight edge tests;
4. Hydraulic aperture (e); and,
5. Mechanical aperture (E).

These properties of samples were measured as part of the field work completed by Cheema et al. (2006) at SSFL. The work provides the basis for computing a site specific stress-closure curve for rocks at SSFL. These properties were measured during tests of both sandstone and shale rock samples taken during coring at SSFL. A detailed methodology can be found in Cheema et al. (2006). The following table shows the results of the fracture characterization.

The Barton-Bandis model is a set of empirical equations that relates rock and fracture properties (JRC, JCS, e, E, and $\sigma_c$) to allow prediction of stress-closure characteristics of fractures. The model has been shown to closely reproduce the physical measurements of stress-closure curves for unfilled interlocked joints with JRC = 5-15; JCS=22-182 MPa; initial apertures of $a_j = 0.10 – 0.60$ mm (Barton and Bakhtar, 1987). The average field parameter values presented in the table fall within these ranges except for the JRC. The average and minimum JRC values are lower than the range tested by Bandis et al. (1983). The lower JRC values reflect a less weathered fracture surface than tested by Bandis et al. (1983). Although the JRC is lower than what was validated the model is still considered applicable for use at SSFL based on applications described by NGI (2006).

A spreadsheet model was developed for this project that allows prediction of closure of a single fracture, and/or a fracture distribution, under effective normal stress, based on methodology described by Barton and Bakhtar (1987). The empirical relations are described briefly below. For details see Cheema et al. (2006).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SSFL Sandstone</th>
<th>SSFL Shale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td>Compressive Strength ($\sigma_c$) MPa</td>
<td>23</td>
<td>140</td>
</tr>
<tr>
<td>Joint Compressive Strength (JCS) MPa</td>
<td>21</td>
<td>126</td>
</tr>
<tr>
<td>Joint Roughness Coefficient JRC (Tilt Test)</td>
<td>1.5</td>
<td>5.7</td>
</tr>
<tr>
<td>JRC (Straight Edge)</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>Hydraulic Aperture (e) micron</td>
<td>10</td>
<td>286</td>
</tr>
<tr>
<td>Mechanical Aperture (Eo) micron</td>
<td>50</td>
<td>400</td>
</tr>
</tbody>
</table>

The work by Bandis et al. (1983) determined a general relation (Eqn. 6) for determining the joint closure ($V_j$) with increasing normal stress ($\sigma_n$) for a given rock type.

$$\sigma_n = \frac{\Delta V_j}{a - b \Delta V_j}$$  (Equation 6 from Bandis et al.)
Where \( a \) and \( b \) are constants for a given rock type. Equation 6 describes a hyperbolic function for \( \Delta V_j \) vs. \( \sigma_n \) relation. In its linear form Equation 6 becomes:

\[
\frac{\Delta V_j}{\sigma_n} = a - b\Delta V_j
\]

(Equation 7 from Bandis et al.)

Bandis et al. indicates that as \( \sigma_n \) increases the \( \Delta V_j \) will tend toward a limiting value \( \frac{a}{b} \)

where:

\[
\frac{a}{b} = \text{asymptote to the hyperbola} = V_m \text{ (max. joint closure)}
\]

When \( V_m \) is reached the joint stiffness (\( K_n \)) will acquire and infinite value. For small changes in normal stress, the change in joint closure will approach zero. Therefore \( a \) represents the reciprocal of the initial normal joint stiffness (\( K_{ni} \)).

The values of \( K_{ni} \) and \( V_m \) can be determined from two additional empirical relations developed by Bandis et al. (1983) as follows:

\[
V_m = A + B(JRC) + C\left(\frac{JCS}{a_j}\right)^D
\]

(Equation 24 from Bandis et al.)

\[
K_{ni} = 0.02\left(\frac{JCS}{a_j}\right) + 1.75JRC - 7
\]

(Equation 25 from Bandis et al.)

The term \( a_j \) refers to the initial aperture, or the aperture under the self-weight of the sample (~0.001 KPa). This is equivalent to the normal stress a fracture at surface would experience where fracture surfaces are not seated or a core sample (see Cheema et al., 2006). Equation 26 provides an empirical equation for estimating initial aperture:

\[
a_j = \frac{JRC}{5}\left(0.2 - \frac{\sigma_c}{JCS} - 0.1\right)
\]

(Equation 26 from Bandis et al.)

In our study measurement of mechanical apertures on cores and hydraulic apertures were used instead of this equation to determine the initial aperture as per Barton and Bakhtar (1987).

The specification of an initial aperture (\( a_i \)) is an important concept. The method of physical measurement of fracture closure used by Bandis et al. (1983) and the empirical
model simulation requires four cycles of stress loading and unloading of samples in order to achieve an \textit{in situ} representation of closure. In the first three cycles the loading and unloading process allows the two surfaces defining a fracture to become seated. The last cycle therefore shows the closure response of an \textit{in situ} fracture with fracture surfaces already seated. To represent this effect the Barton-Bandis model utilizes constants (A, B, C, D) as determined Bandis et al. (1983) for each cycle to simulate this seating process. This method enables correlation of physically measurable fractures (at surface, or from cores) and hydraulic apertures to \textit{in situ} conditions.

The values of $K_n$ and $V_m$ are dependent on $a_i$, JCS, and JRC in decreasing order of importance. JCS and JRC have a separate influence on $V_m$. Fractures with the same JCS will show an increase in $V_m$ with a decrease in JRC. In addition an increase in JCS for samples with the same JRC will decrease $V_m$ (Barton and Bakhtar, 1987). Degree of weathering is inferred by comparison of the fracture JCS to the compressive strength of the rock. Where they are equal no weathering of the fractures has occurred. Therefore a less weathered rock will exhibit a larger $V_m$.

\textbf{Conductivity-Closure Derivation}

Barton and Bakhtar (1987) extended the work by Bandis et al. (1983) by describing the empirical relation of change in fracture conductivity with the increase in stress. The relation is based on the cubic law describing flow in a fracture as being equal to the aperture cubed for fractures up to 250 $\mu$m. At the higher end of this range mechanical aperture is generally considered to be equal to hydraulic aperture ($e$). However as apertures decrease (e.g. under high normal stress) Barton and Bakhtar (1987) indicate that the ratio of true mechanical aperture to hydraulic aperture ($E/e$) increases rapidly. For rock samples with JRC values of 3.5, the $E/e$ ratio exceeds 1 below apertures of approximately 10 $\mu$m. Stress-closure behaviour is governed by the mechanical aperture ($E$). The assumption of $E=e$ is valid when fractures are smooth or wide. When fractures become smaller the JRC can be used to predict the divergence from the assumption that $e=E$.

\[
e = \frac{E^2}{JRC^{2.5}} \quad \text{(Equation 96 from Barton-Bakhtar)}
\]

The assumption that $e=E$ for using the cubic law is shown to be valid by Barton and Bakhtar (1987) where JRC is less than 20 and hydraulic aperture $e$ is less than 1000 $\mu$m (1.0 mm) and JRC is greater than 2.5 and $e$ is greater than 10 $\mu$m (0.01 mm).

Therefore by using the Barton-Bandis model to compute the mechanical aperture under a given normal stress (depth) the fracture permeability (cm$^2$) can be estimated from:

\[
k = \frac{e^2}{12} \quad \text{(Equation 119 from Barton-Bakhtar)}
\]
When the fluid is water the hydraulic conductivity in cm/s is:

\[ K = k (9.8 \times 10^4) \]

**Application of Barton-Bandis Spreadsheet Model to SSFL**

The spreadsheet used in the previous section to compute the conductivity with depth based on mean aperture and probability weighted aperture methods (III and IV was modified to incorporate the Barton-Bandis model of fracture closure with the increase in normal effective stress. The Barton-Barton Bandis model based on Bandis et al. equation 6 replaced the original empirical linear relation derived for the sandstone samples (Bandis et al., 1983).

\[ \Delta V_j = \frac{\sigma_n a}{1 + \sigma_n b} \]  
(rearranged Bandis et al. equation 6)

The model was applied as described in Barton and Bakhtar (1987). The stress-closure curves over four cycles were computed for a range of initial mechanical apertures and the average JRC and JCS values reported by Cheema et al., (2006) for SSFL rock samples. The fourth cycle loading curve was used to compute the residual fracture aperture under a normal stress value between 0 and 10.9 MPa, reflecting depths of 0 to 1000 meters below ground surface. This depth is based on the estimate of effective normal stress described in section 2) and 3) in the previous section.

Field data collected for the SSFL shown in the table above have an average mechanical aperture (E) measured on natural fractures from core samples using a feeler gauge of 150 µm, but ranges from 10 to 450 µm. In contrast the average hydraulic aperture reported at SSFL (Montgomery Watson, 2000) is 107 µm (arithmetic) and 70 µm (geometric) with a range of 10 to 286 µm. The frequency of hydraulic aperture distribution is shown in figure below. This table was developed for FRACTRAN simulations.
Figure 5: Fracture aperture distribution developed for the SSFL FRACTRAN simulations (Montgomery Watson, 2000)

In applying the Barton-Bandis model an initial mechanical aperture $E$ must be specified to compute closure with increasing stress. This initial aperture can be computed from feeler gauge values or more reliably using the apertures determined by hydrologic testing (Barton and Bakhtar, 1987). It was assumed here that the authors were referring to estimating the initial mechanical fracture aperture by assuming that $E = e = a_j$.

For comparison the Barton-Bandis model was applied to the original fracture distribution ($e$) presented in the chart above using the range of apertures used in the previous section for initial apertures $a_j$, where $e = a_j$ using the range of 10 to 540 µm. The probability weighted and mean aperture for each depth computed using the Barton-Bandis model is presented in the following table.

Based on this approach the model predicts an average hydraulic aperture of 15 µm over a depth of 0 to 1000 m. At a depth of 50-90 m, approximate average depth interval of tests, the average hydraulic aperture is 18 µm. These values are roughly an order of magnitude lower than the average values measured on site by hydraulic testing. Therefore the hydraulic aperture measured in the field does not appear to predict the initial aperture correctly.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Prob. Weighted Aperture $E$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.043</td>
</tr>
<tr>
<td>10</td>
<td>0.037</td>
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<tr>
<td>20</td>
<td>0.032</td>
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<tr>
<td>60</td>
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<tr>
<td>70</td>
<td>0.020</td>
</tr>
<tr>
<td>80</td>
<td>0.019</td>
</tr>
<tr>
<td>90</td>
<td>0.018</td>
</tr>
<tr>
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<tr>
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<tr>
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<td>0.006</td>
</tr>
<tr>
<td>450</td>
<td>0.005</td>
</tr>
</tbody>
</table>
If the range of starting aperture \(a_j\) derived from the hydraulic testing is multiplied by a factor of 3 to give a range of 30 to 1,620 µm (0.03 to 1.6 mm) the average hydraulic aperture predicted by the model is 93 µm over a depth of 0 to 1000 m. At a depth of 50-90 m the average hydraulic aperture is 107 µm.

The joint roughness model described by equation (96) and presented by Barton-Bahktar supports the use of a scaling factor of 3 for computing initial mechanical apertures from hydraulic aperture.

Fracture permeability was calculated for the predicted aperture at normal stress (depth) value for the scaled initial apertures. The fracture conductivity was calculated using equation 119 and converted to hydraulic conductivity (cm/s). Bulk hydraulic conductivity was calculated using the fracture porosity range 5e-6 to 1e-4 as reported in the Site Conceptual Model Report (Montgomery Watson, 2000). A fracture porosity of 9e-5 results in bulk conductivity values of 2.46 E-5 cm/s at the 50 meter depth, which is consistent with the average values determined from hydraulic testing.

Figure 6 shows the SSFL sandstone bulk conductivity with depth using the probability weighted estimate of aperture and the mean aperture values. For comparison the previous versions of the conductivity with depth function are also presented on the figure.

The Figure 6 shows the SSFL shale bulk conductivity with depth using the probability weighted estimate of aperture and the mean aperture values. Hydraulic apertures are not available for shale units. Therefore the measured mechanical apertures are used to estimate \(a_j\). The same frequency distribution is used to estimate the probability of a fracture of a given aperture, based on the hydraulic apertures measured on site for sandstone. The resulting decrease in \(k\) with depth is more extreme than for sandstone.

New \(K\) factors are shown in the following table and are compared with previous estimates. The new \(K\) factors based on the SSFL field data indicate that the adjusted literature values developed in previous sections provide a good estimate of the conductivity with depth scaling at SSFL for sandstone. However the probability weighted method based on field data suggests that conductivities could be larger near the surface but decrease more quickly with depth than predicted by the literature values.
### SSFL Field Based - K factors –Sandstone and Shale

<table>
<thead>
<tr>
<th>From Depth</th>
<th>To Depth</th>
<th>Calibrated Depth Factor</th>
<th>Depth Factor</th>
<th>% Higher</th>
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<tr>
<td>0</td>
<td>20</td>
<td>2.5</td>
<td>4.36</td>
<td>336%</td>
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<td>2.5</td>
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<td>154%</td>
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<td>70</td>
<td>1.67</td>
<td>1.67</td>
<td>67%</td>
</tr>
<tr>
<td>70</td>
<td>80</td>
<td>1.18</td>
<td>1.18</td>
<td>39%</td>
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<tr>
<td>80</td>
<td>90</td>
<td>0.93</td>
<td>0.93</td>
<td>33%</td>
</tr>
<tr>
<td>90</td>
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<td>0.82</td>
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<td>36%</td>
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<td>0.57</td>
<td>0.57</td>
<td>42%</td>
</tr>
<tr>
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<td>0.51</td>
<td>0.51</td>
<td>46%</td>
</tr>
<tr>
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<td>150</td>
<td>0.46</td>
<td>0.46</td>
<td>54%</td>
</tr>
<tr>
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<td>0.36</td>
<td>0.36</td>
<td>45%</td>
</tr>
<tr>
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<td>0.22</td>
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<td>10%</td>
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</tr>
<tr>
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<td>0.07</td>
<td>0.07</td>
<td>36%</td>
</tr>
<tr>
<td>350</td>
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<td>0.05</td>
<td>0.05</td>
<td>0%</td>
</tr>
<tr>
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<tr>
<td>500</td>
<td>1000</td>
<td>0.05</td>
<td>0.05</td>
<td>0%</td>
</tr>
</tbody>
</table>
The relation predicts a continued decrease in conductivity with depth. However at a depth of about 350 meters the conductivity of the matrix is achieved for sandstone (1e-6 cm/s; Montgomery Watson, 2000). Therefore the K factors are prescribed such that the K in the Mountain Scale groundwater model is not reduced below the average matrix value.

For shale the predicted decrease in conductivity is more extreme but is less well supported by the field data (Cheema et al. 2006). For current modeling purposes the same K factors developed based on SSFL sandstone will be applied to the conductivity values for shale.

**Assumptions:**
All of the assumptions in the previous section apply to the update of the function with SSFL field data. In addition it is assumed for the purpose of using the function in the Mountain Scale groundwater model that:

1. The Barton-Bandis model is still valid with a JRC value less than 5, which for SSFL sandstone is 3.5;
2. The scaling factor applied to average hydraulic apertures for predicting the initial aperture is reasonable.
3. The hydraulic aperture frequency distribution is valid for both shale and sandstone.
4. The revised K factors can be used to represent the reduction of conductivity of both sandstone and shale.

**Concluding Remarks**

The revised functions improve the representation of the conductivity-depth relation by computing site-specific relations based on field data. The revised relations better represent the physical conditions found at SSFL, however these relations are only a representation of the mean change in conditions with depth (it does not reflect variability at a given depth).

As the USGS study points out, considerable variability in hydraulic conductivity was still found to exist at every depth (see Figure 4 above). The implication is that our application of this relation in the model will be representative of average conditions (this is appropriate for the Mountain Scale Groundwater Flow Model), but it will not allow us to represent the full range of conductivities that occur at any given depth.
References


